Contemplation about gravity

We suppose two bodies in an empty space with their mass **m<<M**

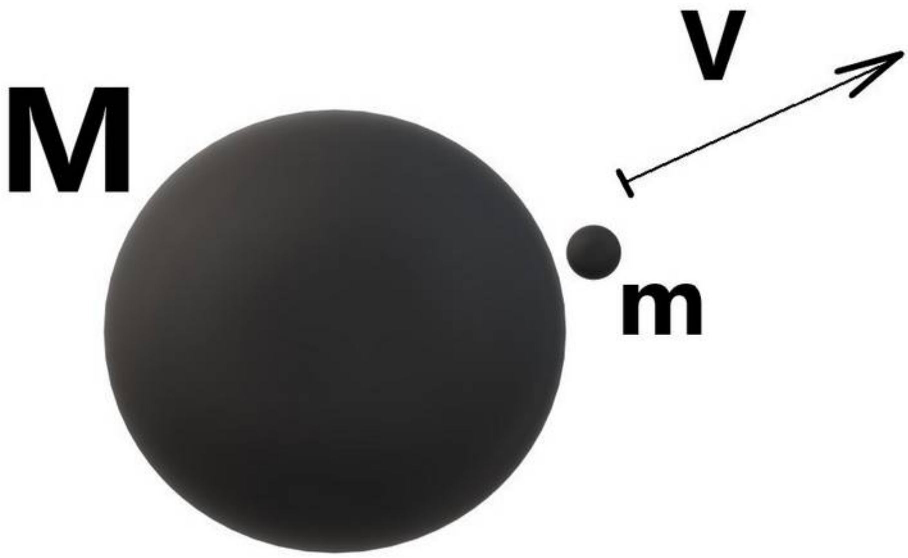

Fig. 1 – two bodies with another mass for a very small distance between them

Kinetic energy E of the moving body m

$$\mathbf{E} = \frac{1}{2}\mathbf{m} * \mathbf{v}^2 \qquad \textbf{[1]}$$

E [J] energy, kinetic to the body M

m [kg] mass of the body m

v [m/s] velocity of the body m to the center of the body M

Potential energy U of the body m in a gravity field of the body M

$$\mathbf{U} = \mathbf{m} * \mathbf{g} * \mathbf{h} \ \textbf{[J]} \qquad \textbf{[2]}$$

m [kg] mass of the body m

g [$\mathbf{m/s^2}$] gravitational acceleration, in our condition mostly $9{,}81 m/s^2$

h [m] height from the surface of the body M

From the law of conservation of energy we know - the change of kinetic energy is equal to the change of potential energy

$\mathbf{\Delta U} = \mathbf{\Delta E}$ better to write $\mathbf{\Delta U = - \Delta E}$ or $\mathbf{\Delta E = - \Delta U}$

Of`course, we must choice the coordinate system and the reference point. The best is use vectors. But in our case we solve only radial motion – the moving only to (or from) the center of gravity.

$$\Delta U = \Delta E \qquad [3]$$

$$mgh_2 - mgh_1 = ½m(v_2)^2 - ½m(v_1)^2 \qquad [3a]$$

$$mg(h_2 - h_1) = ½m[(v_2)^2 - (v_1)^2] \qquad [3b]$$

What does it mean? Fill the equation with a greek` symbol of the difference

$$mg\Delta h = ½m(\Delta v)^2 \qquad [3c]$$

$$g\Delta h = ½(\Delta v)^2 \qquad [3d]$$

$$2g\Delta h = (\Delta v)^2 \qquad [3e]$$

If the velocity grows (+) then the height decreases (-). If the height grows (+) then the velocity decreases (-).

From the equation **[3e]** we obtain

$$\Delta v = \sqrt{2 g\Delta h} \qquad [4]$$

and

$$\Delta h = \frac{\Delta v^2}{(2g)} \qquad [5]$$

Suddenly we could solve next problem. The ball is lying on the ground. The initial velocity is 10 m/s^{-1}. How much the height of the ball will be from the ground? We use the equation **[5]**

$$\Delta h = \frac{10^2}{(2*9.81)} = 5.01\ m$$

In a small range of the distance close to the Earth we suppose g = const. But if we want to solve bigger heights we must use the equation for g. We know that g depends on the distance from the center of the gravity.

$$\mathbf{g} = \mathbf{G} * \left(\frac{\mathbf{M}}{\mathbf{R}^2}\right) \qquad \textbf{[6]}$$

M [kg] mass of the body M e.g. the Earth

G [m/s²] gravitational constant 6,674 3×10^{-11} $m^3 \cdot kg^{-1} \cdot s^{-2}$

R [m] distance (radius) from the center of the gravity of the body M

What about the escape velocity from the Earth? What does it mean? What is the intial velocity of the ball to leave the Earth forever.

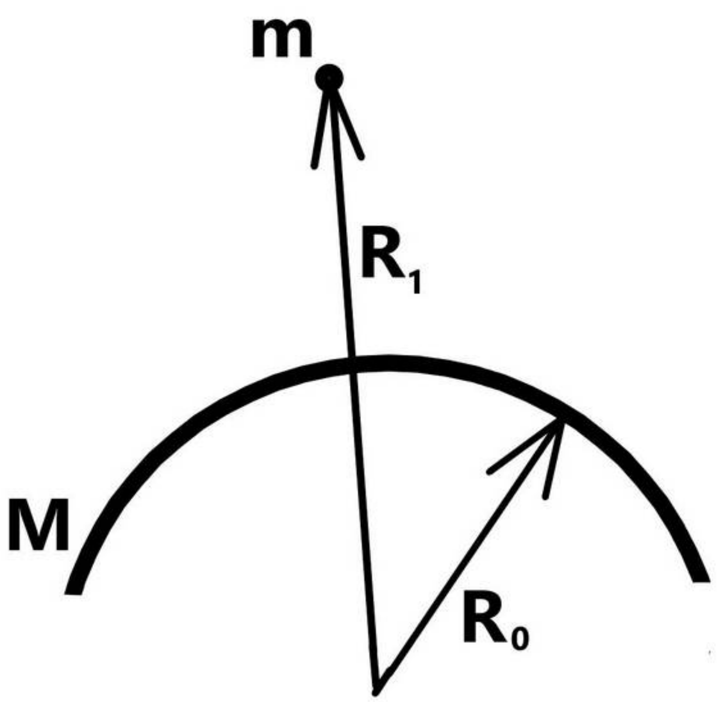

Fig. 2 – dimensions R between two bodies M and m (who wants te leave the Earth)

It means the ball m must have the kinetic energy on the surface of the Earth bigger or equal the potential energy of the ball m on that same place. For the solving we use the equation **[4]**

$$\mathbf{\Delta v} = \sqrt{2.9{,}81.6\,371\,000} \ \mathbf{= 11\ 180{,}296\ m/s}$$

where $\mathbf{\Delta h = R_0}$ we could take the mass of the Earth condensed to its center. The radius of our Earth is 6 371 km then $\mathbf{R_0 = 6\ 371\ 000\ m}$.

O.K. We´v got a right velocity **11,2 km/s.**

If we want to know the escape velocity for a higher radius R then we need to replace constant $g=9.81 m/s^2$ at the eq. **[4]** by the equation **[6]**. Then we obtain

$$\mathbf{v}=\sqrt{2\mathbf{G}*(\frac{\mathbf{M}}{\mathbf{R}^2})\mathbf{R}}=\sqrt{2\mathbf{G}*(\frac{\mathbf{M}}{\mathbf{R}})} \quad \textbf{[7]}$$

We donˇt need anymore the gravitational acceleration, instead we need the mass M = 5,792E24 kg of the Earth and the gravitational constant. After that we obtain the escape velocity from e.g. stratosphere or orbit etc.

We could solve the escape velocity from the surface of the Earth by using the eq. **[7]** either. I´v tried it and obtain **v = 11 016 m/s.** The difference is made by rounded errors.

Anyway - imagine two bodies with the distance e.g. one parsec.

Fig. 3 – two bodies with another mass for a long distance between them

What about energy of such bodies? Go on. We could make a hole inside the big body M. Why? The reason will come later. See the Fig. 4.

Fig. 4 - two globes with another mass for a long distance between them, in the big body M there is a hole through that

Here is a big globe M with a hole through that. The hole passes through the center of such globe. The diameter is even bigger then the diameter of a small globe m.

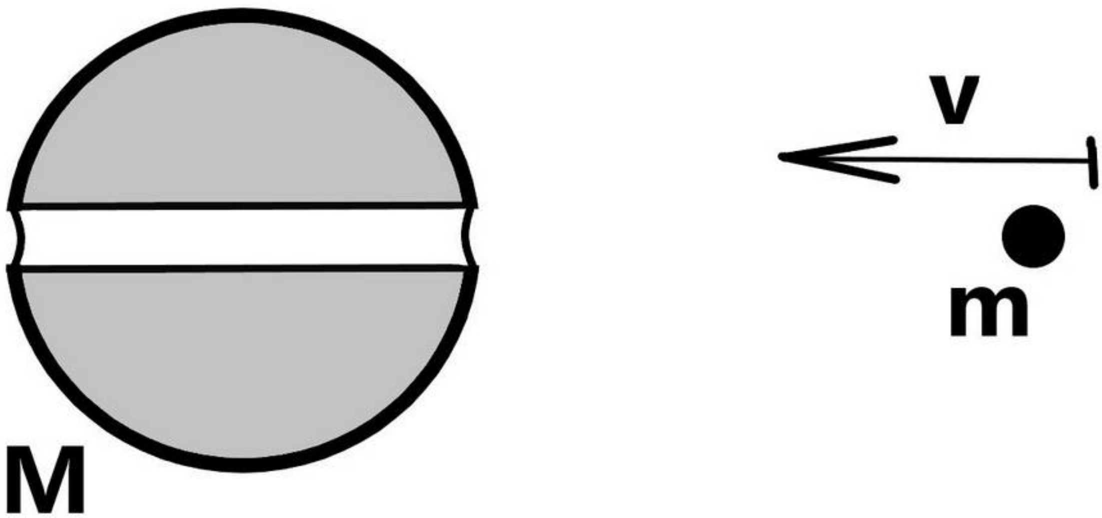

Fig. 5 – a sectional view of Fig. 3

to be continued