The Integral

How to count the area under a function? E.g. under the function sin(x) or anything else. It is a quite difficult question. Especially if we take the square for a base unit of the area. How to count the area of a circle with the square units? The best way is to divide the circle at small squares or rectangles. Then we count how many small rectangles are inside the circle.

In the next figure there is a non-specific function f(x) – If we want to count the area under such function in a range from point x_1 to point x_2 then we must divide the range to the same intervals – dx. There are 12 intervals there. For every interval dx we count the area by using dA=f(x).dx.

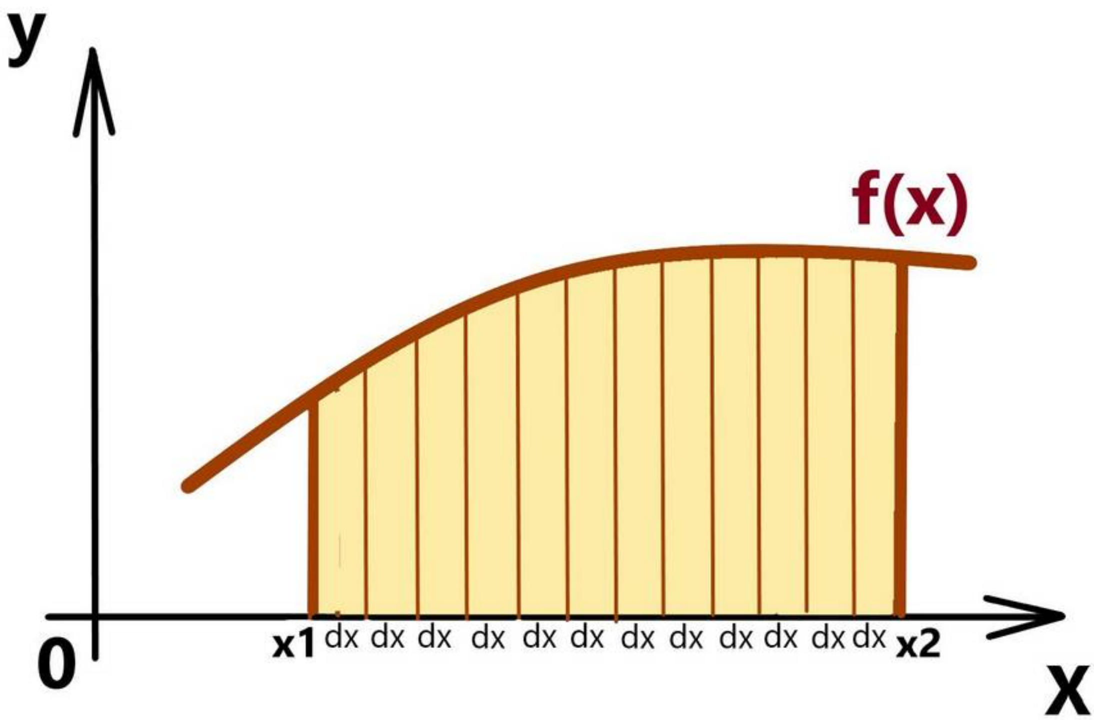

Fig. 1 – the course of a function f(x)

After that we make the sum of all rectangles with a suitable numerical method. This is the area. We could write down

$$\mathbf{A} = \sum_{\mathbf{i}=1}^{12} \mathbf{f}(\mathbf{x_i}) * \mathbf{dx} \qquad \mathbf{[1]}$$

in a better way if dx → 0 there is the integral

$$\mathbf{A} = \int_{\mathbf{x}_1}^{\mathbf{x}_2} \mathbf{f}(\mathbf{x}) * \mathbf{dx} \qquad \textbf{[2]}$$

But we are interested if there is an equation **A (x)** to allow us directly to count the area under the function without to use rectangles (numerical methods).

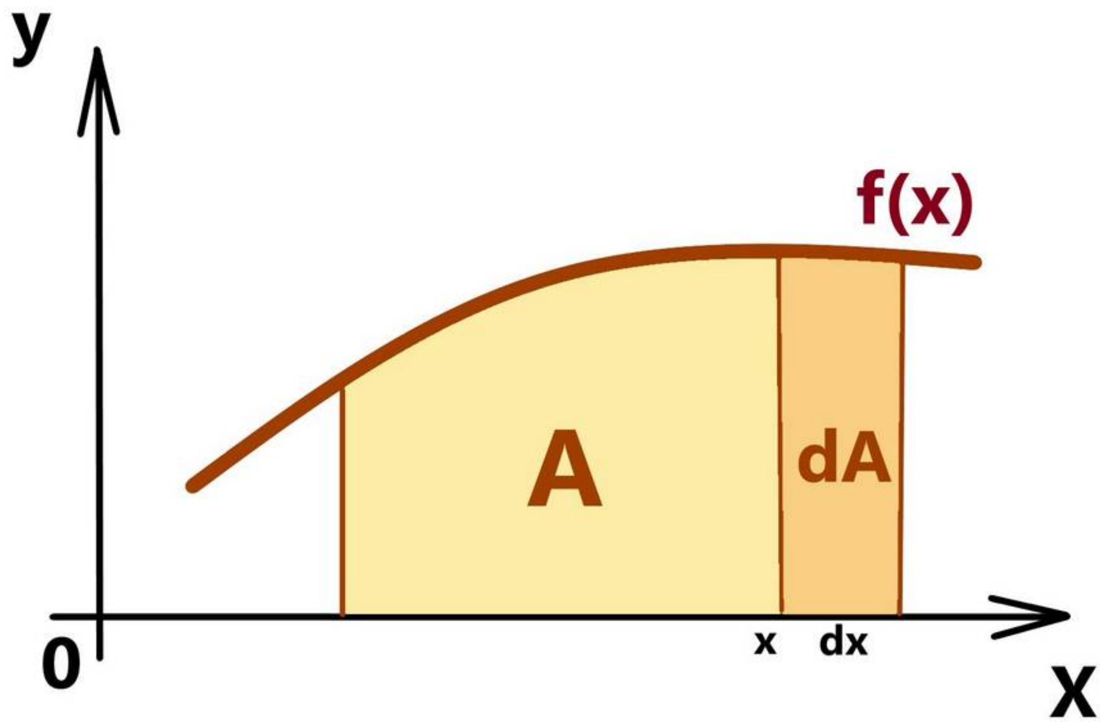

Fig. 2 – the course of a function f(x)

We have in the present time the unknown function **A(x)**
and a very familiar elementary area **dA = f(x).dx**

we could write down from the Fig. 2

$$\mathbf{A(x) + dA = A(x+dx)} \qquad \textbf{[3]}$$

where, as we know **dA = f(x)*dx [2]**

then $$\mathbf{A(x) + f(x)*dx = A(x+dx)} \qquad \textbf{[4]}$$

after a separation of

$$\mathbf{f(x)*dx = A(x+dx) - A(x)}$$

we write down

$$\mathbf{f(x) = \frac{A(x+dx) - A(x)}{dx} = \frac{dA}{dx}} \qquad \textbf{[5]}$$

Nothing new, we only see a very familiar formula for the derivation of the function A(x). The result of the derivation of the function A(x) is the function f(x).The formula A(x) for the counting of the area under the function f(x) is valid for all kind of regular functions f(x), never mind how different are there.

Let´s go for an example

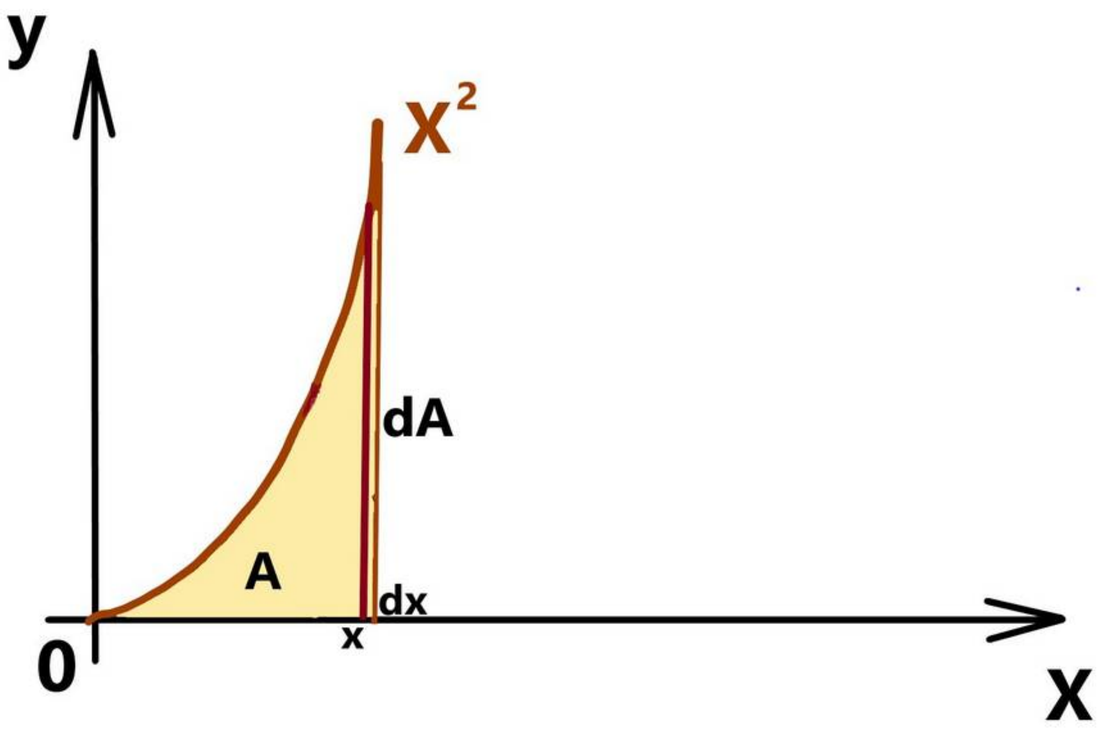

Fig. 3 – function $\mathbf{f(x) = x^2}$

Take out the „curve" of a function $\mathbf{f(x) = x^2}$. We suppose from [5] the function $\mathbf{A}(\mathbf{x})=\frac{1}{3}\cdot\mathbf{x}^2$ is valid. **[6]**

From [5] after substitution for **A(x)**, we get

$$\mathbf{f}(\mathbf{x})=\frac{\frac{1}{3}(\mathbf{x}+\mathbf{dx})^3-\frac{1}{3}(\mathbf{x})^3}{\mathbf{dx}} \quad \textbf{[7]}$$

then

$$\mathbf{f}(\mathbf{x})=\frac{\frac{1}{3}(\mathbf{x}^3+3*\mathbf{x}^2*\mathbf{dx}+3*\mathbf{x}*\mathbf{dx}^2+\mathbf{dx}^3)-\frac{1}{3}\mathbf{x}^3}{\mathbf{dx}}$$

then

$$\mathbf{f}(\mathbf{x})=\frac{\frac{1}{3}\mathbf{x}^3+\mathbf{x}^2*\mathbf{dx}+\mathbf{x}*\mathbf{dx}^3+3*\mathbf{dx}^3-\frac{1}{3}\mathbf{x}^3}{\mathbf{dx}}$$

after reduction, we obtain

$$\mathbf{f(x) = x^2 + xdx + 3dx^3} \quad \textbf{[8]}$$

if dx → 0 then xdx and 3dx = 0, finally we obtain

$$\mathbf{f(x) = x^2} \quad \textbf{[9]}$$

Is it correct? If we put out the members with dx from [7]? We could put out them from [6], either? If dx is close to 0, then in the fraction [5] the member (x+dx) is close to x. Yes, we could say the same about the following equation where x → 0.

$$\left(1+\frac{1}{\mathbf{x}}\right)^{\mathbf{x}}$$

Yes, we know there is the third power. But if we want to derivate [6] again, why we put out the memebers with dx? The power also there is.
Let´s go for next examples.

If we fill the equation [6] with real numbers for x and dx. Firstly for x =2 and for dx=0,001, secondly for x=2 and dx=0,000 001

the dividend dA

$$\mathbf{dA}=\frac{1}{3}(2+0,001)^3-\frac{1}{3}(2)^3$$ = 0,004 002 000 33. ...

if dx → 0 then dA → 0. What about the f(x) at the point 2? Remember, there is a divisor dx. If we multiply 0,0,004 002 000 33 by a number 1/dx then we obtain a real number 4,002 000 33. ... I repeat the real number not an integer.

$$\mathbf{f}(\mathbf{x})=\frac{\frac{1}{3}(2+0,001)^3-\frac{1}{3}(2)^3}{0,001}$$ = 4,002 000 333 333 33

For the next case with x =2 and with dx=0,000 001

$$\mathbf{f}(\mathbf{x})=\frac{\frac{1}{3}(2+0,000001)^3-\frac{1}{3}(2)^3}{0,000001}$$ = 4,000 002 000 000 33

What does it mean? Nothing new, as usual. If we multiply dx by 1/dx we get always the number 1. Our work is to separate this one from other members by using of algebra´ rules. The equation of x^2 [9] is hidden at the equation [7].

to be continued