Contemplation about gravity

We suppose two bodies in an empty space with their mass **m<<M.** In gravitational fields of these bodies there are three forms of motion:

1) a free fall of the body m to the center of a gravity field of the body M

2) to go away from the center of the gravity until some distance and then the form 1), or to go away forever with the escape speed

3) a free fall with some longitunidal speed in such value to be at the same high around the gravity globe, it´s known as an orbit speed 7.8 km/s around the Earth

It is clear such kinds of a gravity motion among sources of gravity is valid for all our universe.

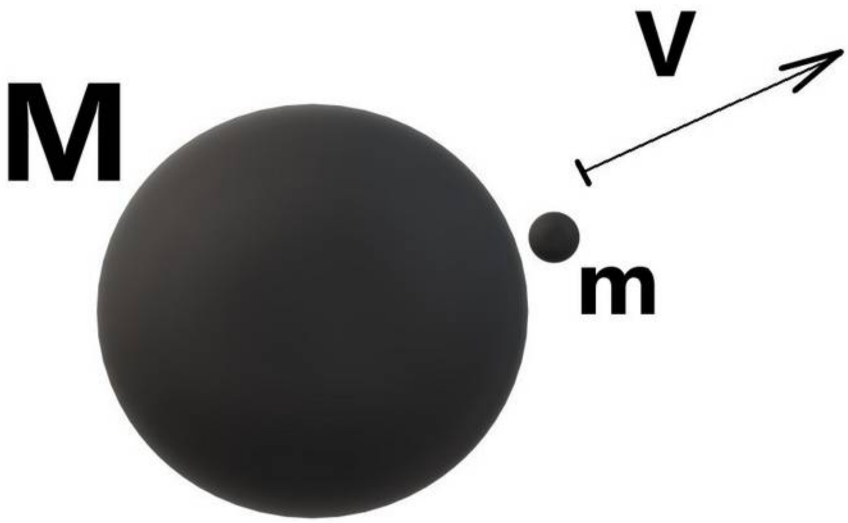

Fig. 1 – two bodies with another mass for a very small distance between them

Kinetic energy E of the moving body m

$$\mathbf{E}=\frac{1}{2}\mathbf{m}*\mathbf{v}^2 \qquad \textbf{[1]}$$

E [J] the kinetic energy of the body m to the body M

m [kg] mass of the body m

v [m/s] the velocity of the body m to the body M

Potential energy U of the body m in a gravity field of the body M

$$\mathbf{U = m * g * h} \text{ [J]} \qquad [2]$$

m [kg] mass of the body m

g [m/s^2] gravitational acceleration, in our condition mostly 9,81m/s^2

h [m] the height from the surface of the body M

From the law of the conservation of energy we know - the change of the kinetic energy is equal to the change of the potential energy

$\mathbf{\Delta U = \Delta E}$ better to write $\mathbf{\Delta U = - \Delta E}$ or $\mathbf{\Delta E = - \Delta U}$

Of`course, we must choice the coordinate system and the reference point. The best is use vectors. But in our case we solve only radial motion – the motion only to (or from) the center of gravity.

$$\mathbf{\Delta U = \Delta E} \qquad [3]$$

$$\mathbf{mgh_2 - mgh_1 = ½m(v_2)^2 - ½m(v_1)^2} \qquad [3a]$$

$$\mathbf{mg(h_2 - h_1) = ½m[(v_2)^2 - (v_1)^2]} \qquad [3b]$$

What does it mean? Fill the equation with a greek` symbol Δ of the difference

$$\mathbf{mg\Delta h = ½m(\Delta v)^2} \qquad [3c]$$

$$\mathbf{g\Delta h = ½(\Delta v)^2} \qquad [3d]$$

$$\mathbf{2g\Delta h = (\Delta v)^2} \qquad [3e]$$

If the velocity increases (+) then the height decreases (-). If the height increases (+) then the velocity decreases (-).

From the equation **[3e]** we obtain

$$\mathbf{\Delta v} = \sqrt{2\mathbf{g\Delta h}} \qquad [4]$$

and

$$\mathbf{\Delta h} = \frac{\mathbf{\Delta v}^2}{(2\mathbf{g})} \qquad [5]$$

Suddenly we could solve next problem. The ball is lying on the ground. The initial velocity is 10 m/s^{-1}. How much the height of the ball will be from the ground? We use the equation **[5]**

$$\mathbf{\Delta h} = \frac{10^2}{(2*9.81)} = \mathbf{5.01\ m}$$

In a small range of the distance close to the Earth we suppose g = const. But if we want to solve larger heights we must use the equation for g. We know that g depends on the distance from the center of the gravity.

$$\mathbf{g} = \mathbf{G} * \left(\frac{\mathbf{M}}{\mathbf{R}^2}\right) \qquad \textbf{[6]}$$

M [kg] mass of the body M (e.g. the Earth)

G [m/s^2] gravitational constant 6,674 3×10^{-11} $m^3 \cdot kg^{-1} \cdot s^{-2}$

R [m] distance (radius) from the center of the gravity of the body M

What about the escape speed from the Earth? What does it mean? What is the intial velocity of the ball to leave the Earth forever.

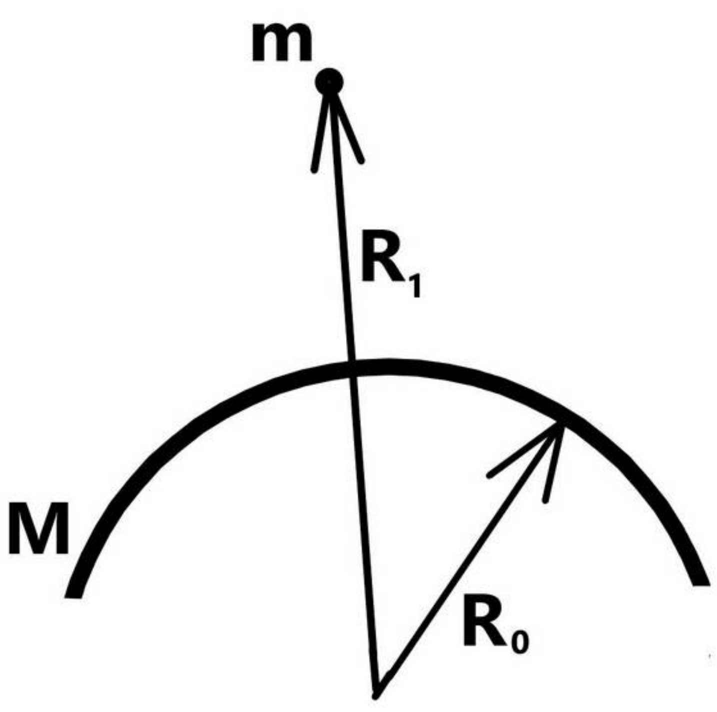

Fig. 2 – dimensions R between two bodies M and m (who wants to leave the Earth)

It means the ball m must have the kinetic energy on the surface of the Earth larger or equal to the potential energy of the ball m on that same place. For the solving that we use the equation **[4]**

$$\mathbf{\Delta v} = \sqrt{2.9,81.6371000} \ \mathbf{= 11\ 180{,}296\ m/s}$$

where $\mathbf{\Delta h = R_0}$ we could take the mass of the Earth condensed to its center. The radius of our Earth is 6 371 km then $\mathbf{R_0 = 6\ 371\ 000\ m}$.

O.K. We´v got a right velocity **11,2 km/s.**

If we want to know the escape velocity for a higher radius R then we need to replace constant g=9.81m/s^2 at the eq. **[4]** by the equation **[6]**. Then we obtain

$$\mathbf{v} = \sqrt{2\mathbf{G} * (\frac{\mathbf{M}}{\mathbf{R}^2})\mathbf{R}} = \sqrt{2\mathbf{G} * (\frac{\mathbf{M}}{\mathbf{R}})} \qquad \textbf{[7]}$$

We don´t need anymore the gravitational acceleration, instead we need the mass M = 5,792E24 kg of the Earth and the gravitational constant. After that we obtain the escape velocity from e.g. stratosphere or orbit etc.

We could solve the escape velocity from the surface of the Earth with the eq. **[7]** either. I´v tried it and obtain **v = 11 016 m/s.** The difference is made by rounded errors.

Such velocity enables of a ball m to escape from a mass M into infinity. But in our universe there are not only two bodies. There are plenty planets, stars, galaxies, metagalaxies. If the body m obtain the escape velocity 11,2 km/s from the Earth it does not mean the body m will escape to infinity. The reason is our solar system with the Sun and planets. The ball with velocity 11,2 km/s forever leaves the Earth, but become a new smallest planet of our solar system. To leave our solar system the ball needs the velocity 16 km/s (at the distance of our Earth from the Sun). If the ball obtains such velocity then the ball leaves forever our solar system. But the troubles with the escaping of the ball to infinity does not end. The ball will leave forever our solar system, but never leaves our galaxy (a Milky way). The ball must be accelerated to higher velocity (try it solve with using of a mass of our galaxy and with the radius of our solar system from the center of our galaxy). If the ball obtains the escape velocity from our galaxy then the ball will escape among other galaxies (metagalaxies). To leave metagalaxies the ball must be accelerated to more higher velocity. Finally for the ball to escape our universe (with planty metagalaxies) the ball needs the velocity of the light?

Anyway - imagine two bodies alone with the distance one parsec. It means – two bodies M and m which are alone in the empty infinity space. see Fig 3.

Fig. 3 – two bodies with another mass for a long distance between them

If we solve the escape velocity for two bodies (m and M) then it does not mean they will leave themselves to the infinity. Why not? Because we solved the Euclidean geometry and today as we know every mass deforms a space arround it. Such bodies with the escape velocities only will go around a very big circle. Maybe such circle could be more then 100 times bigger then a diameter of our universe.

Anyway, let´s go to our case. How big is the work to translate the body m to the infinity from the body M.

The equation for the work A is

$$A = F \,.\, \Delta R \qquad [8]$$

A [J]	the mechanical work
F [N]	the force during the travel
ΔR [m]	the travel distance (radius)

What about the force F?

$$F = m \,.\, g \qquad [9]$$

F [N]	the force
m [kg]	the weight of the body m
g [m/s²]	gravitational acceleration, in our condition mostly 9,81m/s^2

But we know the gravitational constant depends on the distance (radius) from the big body M. See the eq. **[6].**

We replace the „constant" g at eq.**[8]** by equation **[6]** and we obtain

$$\mathbf{F = m\ .\ \ G}*\left(\frac{\mathbf{M}}{\mathbf{R}^2}\right) \qquad [10]$$

Now, we know the gravitational force as the function of the radius

Easily we obtain the equation for the work

$$\mathbf{A = m\ .\ G}*\left(\frac{\mathbf{M}}{\mathbf{R}^2}\right)\ \mathbf{.\ \Delta R} \qquad [11]$$

For the reason of the change of the force during the travel (by the changing of g) we must use a differential equation – we write the elementary work **dA** for the elementary motion along **dR**. There is a const. g, in the end we add all elementary works dA with another g through the Δ R.

$$\mathbf{dA = m\ .\ G}*\left(\frac{\mathbf{M}}{\mathbf{R}^2}\right)\ \mathbf{.\ dR} \qquad [12]$$

the substitution **C = m.M.G**

$$\mathbf{dA = C}\ \left(\frac{1}{\mathbf{R}^2}\right)\ \mathbf{.\ dR} \qquad [13]$$

after integration we obtain

$$\mathbf{A = C}\ \int_{\mathbf{R}1}^{\mathbf{R}2}\left(\frac{1}{\mathbf{R}^2}\right)\ \mathbf{.\ dR} \qquad [14]$$

$$\mathbf{A = C}\ \left[-\left(\frac{1}{\mathbf{R}_2}\right)-\left(\frac{1}{\mathbf{R}_1}\right)\right]\ \ \mathbf{=\ C}\ \left[-\left(\frac{1}{\mathbf{R}_2}\right)+\left(\frac{1}{\mathbf{R}_1}\right)\right]$$

then from **C = m.M.G**

$$\mathbf{A =\ G.m.M}\ \left[\left(\frac{1}{\mathbf{R}_1}\right)-\left(\frac{1}{\mathbf{R}_2}\right)\right] \qquad [15]$$

We obtain a suitable formula for the work between $\mathbf{R_1}$ and $\mathbf{R_2}$. If we put the $R_2 = \infty$ then we get

$$\mathbf{A =\ G.m.M}\ \left(\frac{1}{\mathbf{R}_1}\right) \qquad [16]$$

Go on. We could make a hole inside the big body M. Why? The reason will come later. See the Fig. 4.

Fig. 4 - two globes with another mass for a long distance between them, in the big body M there is a hole through that

The reason is to generate an oscillating motion with the two mass. As we see with the spring.

To say something to the gravity. If we have two balls m and M close to each other then we have some gravitational energy. If we the distance between two balls go up then the gravitational energy go up (we need the energy from somewhere to supply such process). But if we allowed to press two balls to each other what about the energy? The energy increase by the pressing of the balls. See Einsteins´ thinking experiment with a closed tube with ELMG waves inside it.

Here is a big globe M with a hole through that. The hole passes through the center of such globe. T

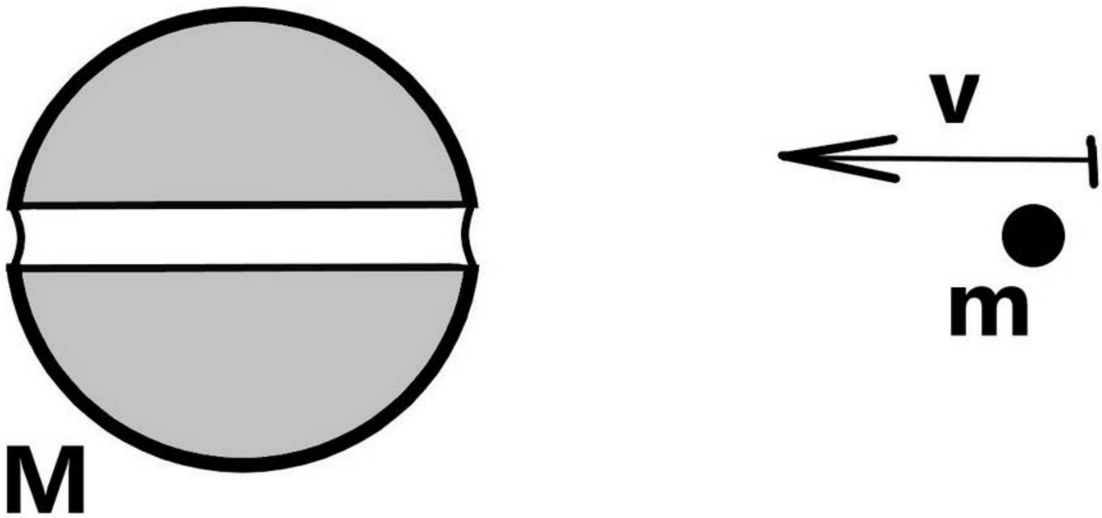

Fig. 5 – a sectional view of Fig. 3

We have an escape velocity close to c (299 792 458 m/s).

It is quite difficult to estimate the behavior of the black hole. We know if we press the ELMG waves then their frequency increase rapidly. The same thinking experiment made by A. Einstein with a „parcel" of ELMG waves bounds in closed space. If we press the „parcel" its energy increase. For some time the energy could be more then the energy of our universe.

to be continued